JEE MAIN + ADVANCED

MATHEMATICS

TOPIC NAME

PROPERTIES

OF

TRIANGLE

(PRACTICE SHEET)

LEVEL- 1

Question based on Sine and Cosine rule

Q.1 In Δ ABC, a = 4, b = 12 and B = 60º then the value of sin A is-

(A) $\frac{1}{2\sqrt{3}}$ (B) $\frac{1}{3\sqrt{2}}$ (C) $\frac{2}{\sqrt{3}}$ (D) $\frac{\sqrt{3}}{2}$

Q.2 Let ABC be a triangle such that ∠A = 45º, ∠B = 75º then a + c $\sqrt{2}$ is equal to-

(A) 0 (B) b (C) 2b (D) –b

Q.3 In ΔABC, 2(bc cosA + ca cosB + ab cosC) =

(A) 0 (B) a + b + c

(C) $a^2 + b^2 + c^2$ (D) None of these

Q.4 In ΔABC,

(b – c) sin A + (c – a) sin B + (a – b) sin C =

(A) ab + bc + ca (B) $a^2 + b^2 + c^2$

(C) 0 (D) None of these

Q.5 If in a Δ ABC, a sin A = b sin B, then the triangle is-

(A) isosceles (B) right angled

(C) equilateral (D) none of these

Q.6 In any Δ ABC if 2 cos B = $\frac{a}{c}$, then the triangle is -

(A) right angled (B) equilateral

(C) isosceles (D) none of these

Q.7 The straight roads of intersect at an angle of 60º. A bus on one road is 2 km away from the intersection and a car on the other road is 3 km away from the intersection. Then the direct distance between the two vehicles is-

(A) 1 km (B) $\sqrt{2}$ km

(C) 4 km (D) $\sqrt{7}$ km

Q.8 If a = 9, b = 8 and c = x satisfies 3 cos C = 2, then-

(A) x = 5 (B) x = 6

(C) x = 4 (D) x = 7

Q.9 In a triangle ABC, sin A : sin B : sin C = 1 : 2 : 3. If b = 4 cm, then the perimeter of the triangle is

(A) 6 cm (B) 24 cm

(C) 12 cm (D) 8 cm

Question based on Napier and Projection Rule

Q.10 In any Δ ABC,

2 [bc cosA + ca cosB + ab cosC] =

(A) $a^2 + b^2 + c^2$ (B) $a^2 – b^2 + c^2$

(C) $a^2 + b^2 – c^2$ (D) $a^2 – b^2 – c^2$

Q.11 In a Δ ABC, if A = 30º, b = 2, c = $\sqrt{3}$ +1, then $\frac{C-B}{2}$ =

(A) 15º (B) 30º

(C) 45º (D) None of these

Q.12 In Δ ABC, if a cos A = b cos B, then the triangle is-

(A) Isosceles

(B) Right angled

(C) Isosceles or right angled

(D) Right angled isosceles

Q.13 In a triangle ABC,

(b + c) cosA + (c + a) cosB + (a + b) cosC

(A) 0 (B) 1

(C) a + b + c (D) 2(a + b + c)

Q.14 cot $\frac{A+B}{2}$ .tan $\frac{A-B}{2}$ =

(A) $\frac{a+b}{a-b}$ (B) $\frac{a-b}{a+b}$

(C) $\frac{a}{a+b}$ (D) None of these

Question based on Half angle Formulae

Q.15 In any Δ ABC, $\left(\frac{b-c}{a}\right)\cos^2\left(\frac{A}{2}\right)$ + $\left(\frac{c-a}{b}\right)\cos^2\left(\frac{B}{2}\right) + \left(\frac{a-b}{2}\right)\cos^2\left(\frac{C}{2}\right)$ =

(A) 2 (B) 0

(C) 1 (D) None of these

Q.16 $bc\cos^2\frac{A}{2} + ca\cos^2\frac{B}{2} + ab\cos^2\frac{C}{2} =$

(A) $(s-a)^2$ (B) $(s-b)^2$

(C) $(s-c)^2$ (D) s^2

Q.17 In a ΔABC, $s\left[\tan\left(\frac{A}{2}\right)+\tan\left(\frac{B}{2}\right)\right]$ is equal to

(A) $\frac{ab}{R}$ (B) $\frac{2ab}{\Delta}$

(C) $c\cot\left(\frac{C}{2}\right)$ (D) None of these

Q.18 In triangle ABC, $\tan\frac{A}{2} : \tan\frac{B}{2} =$

(A) (s – b) : (s – c) (B) s : (s – c)

(C) (s – a) : (s – b) (D) (s – b) : (s – a)

Question based on Area of triangle

Q.19 In a Δ ABC, (b + c – a) $\tan\left(\frac{A}{2}\right)$ is equal to-

(A) $\frac{2\Delta}{s}$ (B) $\frac{\Delta}{s}$ (C) $\frac{\Delta s}{bc}$ (D) $\frac{s}{a}R$

Q.20 In a ΔABC, if a = 2x, b = 2y and $\angle C = 120^o$, then the area of the triangle is-

(A) xy (B) $xy\sqrt{3}$

(C) 3xy (D) 2xy

Question based on Circumcircle & Radius

Q.21 In a ΔABC, $2R^2 \sin A \sin B \sin C =$

(A) Δ (B) 2Δ

(C) $\Delta / 2$ (D) None of these

Q.22 If the lengths of the sides of a triangle are 3, 4 and 5 units then R the circumradius is-

(A) 2.0 (B) 2.5 (C) 3.0 (D) 3.5

Q.23 In an equilateral triangle of side $2\sqrt{3}$ cms. The circumradius is-

(A) 1 cm (B) $\sqrt{3}$ cm

(C) 2 cm (D) $2\sqrt{3}$ cm.

Question based on Incircle & Inradius

Q.24 In any ΔABC, cos A + cos B + cos C =

(A) $\left(1+\frac{r}{R}\right)$ (B) $\left(1-\frac{r}{R}\right)$

(C) $\left(1+\frac{R}{r}\right)$ (D) None of these

Q.25 In a triangle a = 13, b = 14, c = 15, r =

(A) 4 (B) 8

(C) 2 (D) 6

Q.26 In a triangle ABC, $\frac{a\cos A + b\cos B + c\cos C}{a+b+c}$ is equal to-

(A) $\frac{r}{R}$ (B) $\frac{R}{r}$ (C) $\frac{2r}{R}$ (D) $\frac{R}{2r}$

Q.27 If the sides of a triangle are 3 : 7 : 8 then R : r =

(A) 2 : 7 (B) 7 : 2 (C) 3 : 7 (D) 7 : 3

Q.28 In an equilateral triangle the inradius and the circumradius are consected by-

(A) r = 4R (B) r = R/2

(C) r = R/3 (D) None of these

Q.29 The inradius of the triangle whose sides are 3, 5, 6, is-

(A) $\sqrt{\frac{8}{7}}$ (B) $\sqrt{8}$ (C) $\sqrt{7}$ (D) $\sqrt{\frac{7}{8}}$

Question based on r_1, r_2 & r_3

Q.30 In an equilateral triangle, the in-radius, circum-radius and one of the ex- radii are in the ratio-

(A) 2 : 3 : 5 (B) 1 : 2 : 3

(C) 1 : 3 : 7 (D) 3 : 7 : 9

Q.31 $r_2 r_3 + r_3 r_1 + r_1 r_2 =$

(A) s^2 (B) s (C) s/r^3 (D) R^2

Q.32 $(r_1 + r_2)(r_2 + r_3)(r_3 + r_1) =$

(A) Rs^2 (B) $2Rs^2$

(C) $3Rs^2$ (D) $4Rs^2$

Q.33 If $r_1 = r_2 + r_3 + r$, then the Δ is-

(A) Equilateral (B) Isosceles

(C) Right angled (D) None of these

Q.34 If $\frac{r}{r_1} = \frac{r_2}{r_3}$, then-

(A) A = 90° (B) B = 90°

(C) C = 90° (D) None of these

LEVEL- 2

Q.1 If in a triangle the angles are in A.P. and b : c = $\sqrt{3}$: $\sqrt{2}$, then $\angle$ A is equal to -

(A) 30º (B) 60º
(C) 15º (D) 75º

Q.2 In Δ ABC, if $\sin^2A + \sin^2B = \sin^2C$, then the triangle is -

(A) Equilateral (B) Isosceles
(C) Right angled (D) None of these

Q.3 If in a Δ ABC, $\cos A = \frac{\sin B}{2\sin C}$, then the ΔABC is -

(A) Equilateral (B) Isosceles
(C) Right angled (D) None of these

Q.4 If $c^2 = a^2 + b^2$, then $4s(s-a)(s-b)(s-c) =$

(A) s^4 (B) b^2c^2
(C) c^2a^2 (D) a^2b^2

Q.5 $\frac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B} =$

(A) $\frac{a^2+b^2}{a^2+c^2}$ (B) $\frac{b^2+c^2}{b^2-c^2}$
(C) $\frac{c^2-a^2}{a^2+b^2}$ (D) None of these

Q.6 $r\,r_1 + r_2\,r_3 =$

(A) ba (B) ac
(C) bc (D) abc

Q.7 $r_1 + r_2 =$

(A) $c\tan\left(\frac{C}{2}\right)$ (B) $c\cot\left(\frac{C}{2}\right)$
(C) $c\sin\left(\frac{C}{2}\right)$ (D) $c\cos\left(\frac{C}{2}\right)$

Q.8 $16R^2\,r\,r_1\,r_2\,r_3 =$

(A) abc (B) $a^3\,b^3\,c^3$
(C) $a^2\,b^2\,c^2$ (D) $a^2\,b^3\,c^4$

Q.9 In ΔABC, $a\sin(B-C) + b\sin(C-A) + c\sin(A-B) =$

(A) 0 (B) a + b + c
(C) $a^2 + b^2 + c^2$ (D) $2(a^2 + b^2 + c^2)$

Q.10 In a ΔABC, if a = 8, b = 15, c = 17 then $\sin\frac{A}{2}$ and cos A are equal to-

(A) $\frac{1}{\sqrt{17}}, \frac{15}{17}$ (B) $\frac{2}{\sqrt{17}}, \frac{13}{17}$
(C) $\frac{2}{\sqrt{17}}, \frac{11}{17}$ (D) None of these

Q.11 In any ΔABC, 4Δ(cotA + cotB + cotC) is equal to -

(A) $3(a^2 + b^2 + c^2)$ (B) $2(a^2 + b^2 + c^2)$
(C) $(a^2 + b^2 + c^2)$ (D) None of these

Q.12 If the sides of a triangle are proportional to the cosine of the opposite angles, then the triangle is-

(A) Right angled (B) equilateral
(C) obtuse angled (D) None of these

Q.13 In a triangle ABC, $(a + b + c)(b + c - a) = \lambda bc$ if -

(A) $\lambda < 0$ (B) $\lambda > 0$
(C) $0 < \lambda < 4$ (D) $\lambda > 4$

Q.14 In ΔABC, if $(a + b + c)(a - b + c) = 3ac$, then -

(A) $\angle B = 60º$
(B) $\angle B = 30º$
(C) $\angle C = 60º$
(D) $\angle A + \angle C = 90º$

Q.15 In a triangle ABC, if $b^2 + c^2 = 3a^2$, then cot B + cot C – cot A is equals to -

(A) 1 (B) $\frac{ab}{4\Delta}$ (C) 0 (D) $\frac{ac}{4\Delta}$

Q.16 In ΔABC, if 2s = a + b + c, then the value of $\frac{s(s-a)}{bc} - \frac{(s-b)(s-c)}{bc} =$

(A) sin A (B) cos A
(C) tan A (D) None of these

LEVEL- 3

Q.1 If the median of ΔABC through A is perpendicular to AB, then-
(A) $\tan A + \tan B = 0$ (B) $2\tan A + \tan B = 0$
(C) $\tan A + 2\tan B = 0$ (D) None of these

Q.2 In a ΔABC, if $r = r_2 + r_3 - r_1$, and $\angle A > \frac{\pi}{3}$ then the range of $\frac{s}{a}$ is equal to-
(A) $\left(\frac{1}{2}, 2\right)$ (B) $\left(\frac{1}{2}, \infty\right)$
(C) $\left(\frac{1}{2}, 3\right)$ (D) $(3, \infty)$

Q.3 If in a triangle ABC,
$\cos A \cos B + \sin A \sin B \sin C = 1$,
then the sides are proportional to-
(A) $1 : 1 : \sqrt{2}$ (B) $1 : \sqrt{2} : 1$
(C) $\sqrt{2} : 1 : 1$ (D) None of these

Q.4 If λ be the perimeter of the ΔABC then $b\cos^2\frac{C}{2} + c\cos^2\frac{B}{2}$ is equal to-
(A) λ (B) 2λ
(C) $\lambda/2$ (D) None of these

Q.5 In any triangle ABC, $\sum\frac{\sin^2 A + \sin A + 1}{\sin A}$ is always greater than-
(A) 9 (B) 3
(C) 27 (D) None of these

Q.6 In a ΔABC, $a\cot A + b\cot B + c\cot C =$
(A) $r + R$ (B) $r - R$
(C) $2(r + R)$ (D) $2(r - R)$

Q.7 If in a ΔABC, $3a = b + c$ then $\tan\frac{B}{2} . \tan\frac{C}{2}$ is equal to-
(A) $\tan\frac{A}{2}$ (B) 1
(C) 2 (D) None of these

Q.8 The equation $ax^2 + bx + c = 0$, where a, b, c are the sides of a ΔABC and the equation $x^2 + \sqrt{2}x + 1 = 0$ have a common root. The measure of $\angle C$ is-
(A) 90° (B) 45°
(C) 60° (D) None of these

Q.9 In a ΔABC, $(c + a + b)(a + b - c) = ab$. The measure of $\angle C$ is-
(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$
(C) $\frac{2\pi}{3}$ (D) None of these

Q.10 The diameter of the circumcircle of a triangle with sides 5 cm, 6 cm and 7 cm is-
(A) $\frac{3\sqrt{6}}{2}$ cm (B) $2\sqrt{6}$ cm
(C) $\frac{35}{48}$ cm (D) None of these

Q.11 Let A, B and C are the angles of a triangle and $\tan\left(\frac{A}{2}\right) = \frac{1}{3}$, $\tan\left(\frac{B}{2}\right) = \frac{2}{3}$. Then $\tan\left(\frac{C}{2}\right)$ is equal to-
(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{2}{9}$ (D) $\frac{7}{9}$

Q.12 If A, A_1, A_2, A_3 be the area of the incircle and excircles then $\frac{1}{\sqrt{A_1}} + \frac{1}{\sqrt{A_2}} + \frac{1}{\sqrt{A_3}}$ is equal to-
(A) $\frac{1}{\sqrt{A}}$ (B) $\frac{2}{\sqrt{A}}$
(C) $\frac{3}{\sqrt{A}}$ (D) None of these

Q.13 If α, β, γ are the altitudes of a ΔABC and 2s denotes its perimeter, then $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$ is equal to-
(A) $\frac{\Delta}{s}$ (B) $\frac{s}{\Delta}$
(C) $s.\Delta$ (D) None of these

Q.14 If the perpendicular AD divides the base of the ΔABC such that BD, CD and AD are in ratio 2 : 3 : 6, then angle A is equal to-

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

Q.15 Two sides of a triangle are given by the roots of the equation $x^2 - 2\sqrt{3}x + 2 = 0$. The angle between the sides is $\frac{\pi}{3}$. The perimeter of the triangle is-

(A) $6 + \sqrt{3}$ (B) $2\sqrt{3} + \sqrt{6}$

(C) $2\sqrt{6} + \sqrt{10}$ (D) None of these

Q.16 In a ΔABC, if $3 \tan\frac{A}{2}\tan\frac{C}{2} = 1$, then sides a, b, c are in-

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Q.17 In ΔABC, if a = 16, b = 24, c = 20, then $\sin\frac{A}{2}$ is equal to -

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{\sqrt{2}}$

(C) $\frac{3}{2\sqrt{2}}$ (D) None of these

Q.18 With usual notations, in a ΔABC,

$\frac{b^2 - c^2}{a\sec C} + \frac{c^2 - a^2}{b\sec C} + \frac{a^2 - b^2}{c\sec C}$ is equal to-

(A) 1 (B) 0

(C) abc (D) None of these

Q.19 In an equilateral triangle-

(A) $r_1 = r_2 = r_3 = 2r$ (B) $r_1 = r_2 = r_3 = r$

(C) $r_1 = r_2 = r_3 = 3r$ (D) None of these

LEVEL- 4

(Question asked in previous AIEEE and IIT-JEE)

SECTION –A

Q.1 The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side a, is- **[AIEEE 2003]**

(A) $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$ (B) $a\cot\left(\frac{\pi}{n}\right)$

(C) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ (D) $a\cot\left(\frac{\pi}{2n}\right)$

Q.2 If in a triangle ABC,

$a\cos^2\left(\frac{C}{2}\right) + c\cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$, then the sides a, b and c **[AIEEE 2003]**

(A) satisfy a + b = c (B) are in A.P.

(C) are in G.P. (D) are in H.P.

Q.3 The sides of a triangle are sin α, cos α and $\sqrt{1+\sin\alpha\cos\alpha}$ for some $0 < \alpha < \frac{\pi}{2}$. Then the greatest angle of the triangle is- **[AIEEE 2004]**

(A) 60° (B) 90°

(C) 120° (D) 150°

Q.4 In a triangle ABC, let $\angle C = \frac{\pi}{2}$. If r is the in-radius and R is the circumradius of the triangle ABC, then 2(r + R) equals- **[AIEEE-2005]**

(A) b + c (B) a + b

(C) a + b + c (D) c + a

Q.5 For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is- **[AIEEE 2010]**

(A) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$

(B) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

(C) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

(D) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

Q.6 ABCD is a trapezium such that AB and CD are parallel and BC ⊥ CD. If ∠ADB = θ, BC = p and CD = q, then AB is equal to – **[JEE Main- 2013]**

(A) $\frac{p^2+q^2}{p^2\cos\theta+q^2\sin\theta}$ (B) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)^2}$

(C) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$ (D) $\frac{p^2+q^2\cos\theta}{p\cos\theta+q\sin\theta}$

SECTION –B

Q.1 If in a triangle ABC,

$\frac{2\cos A}{a}+\frac{\cos B}{b}+\frac{2\cos C}{c} = \frac{a}{bc}+\frac{b}{ca}$ then the value of the angle A. **[IIT-1993]**

(A) π/3 (B) π (C) π/2 (D) π/6

Q.2 In a ΔABC, if $\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}$ and the side a = 2, then area of the triangle is- **[IIT-1993]**

(A) 1 (B) 2 (C) $\frac{\sqrt{3}}{2}$ (D) $\sqrt{3}$

Q.3 The sides of a triangle inscribed in a given circle subtend angles α, β, γ at the centre. The minimum value of the A. M. of $\cos\left(\alpha+\frac{\pi}{2}\right)$, $\cos\left(\beta+\frac{\pi}{2}\right)$ and $\cos\left(\gamma+\frac{\pi}{2}\right)$ is equal to- **[IIT-1994]**

(A) $\frac{\sqrt{3}}{2}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $-\frac{2}{\sqrt{3}}$ (D) $\sqrt{2}$

Q.4 In a triangle ABC, $\angle B = \frac{\pi}{3}$ and $\angle C = \frac{\pi}{4}$, Let D divide BC internally in the ratio 1 : 3. Then $\frac{\sin\angle BAD}{\sin\angle CAD}$ equal to- **[IIT-1995]**

(A) $\frac{1}{\sqrt{6}}$ (B) $\frac{1}{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\sqrt{\frac{2}{3}}$

Q.5 If in a triangle PQR, sin P, sin Q and sin R are in A.P., then **[IIT-1998]**

(A) the altitudes are in A.P.

(B) the altitudes are in H.P.

(C) the medians are in G.P.

(D) the medians are in A.P.

Q.6 If the radius of circumcircle of an isosceles triangle PQR is equal to PQ (= PR), then the angle P is- **[IIT(s)1998]**

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

Q.7 If the vertices P, Q, R of a triangle PQR and rational points, then which of the following points of the triangle PQR is (are) always rational point (s) ? **[IIT-1998]**

(A) Centroid (B) Incentre

(C) Circumcentre (D) Orthocentre

Q.8 In a triangle PQR, $\angle R = \frac{\pi}{2}$.

If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0 (a \neq 0)$, then **[IIT-1999]**

(A) a + b = c (B) b + c = a

(C) a + c = b (D) b = c

Q.9 In a ΔABC, $2ac \sin\left(\frac{A-B+C}{2}\right) =$ **[IIT-2000]**

(A) $a^2 + b^2 - c^2$ (B) $c^2 + a^2 - b^2$

(C) $b^2 - c^2 - a^2$ (D) $c^2 - a^2 - b^2$

Q.10 If the angles of a triangle are in ratio 4 : 1 : 1 then the ratio of the longest side and perimeter of triangle is : **[IIT Scr.2003]**

(A) $\frac{1}{2+\sqrt{3}}$ (B) $\frac{2}{\sqrt{3}-2}$

(C) $\frac{\sqrt{3}}{2+\sqrt{3}}$ (D) none of these

Q.11 If the sides a, b, c of a triangle are such that a : b : c : : 1 : $\sqrt{3}$: 2, then A : B : C is- **[IIT Scr.2004]**

(A) 3 : 2 : 1 (B) 3 : 1 : 2

(C) 1 : 3 : 2 (D) 1 : 2 : 3

Q.12 In any Δ ABC having sides a, b, c opposite to angles A, B, C respectively, then- **[IIT Scr.2005]**

(A) $a \sin\left(\frac{B-C}{2}\right) = (b - c) \cos\frac{A}{2}$

(B) $a \cos\frac{A}{2} = (b - c) \sin\frac{B-C}{2}$

(C) $a \cos\frac{A}{2} = (b + c) \sin\frac{B+C}{2}$

(D) $a \sin\frac{B+C}{2} = (b + c) \cos\frac{A}{2}$

Q.13 A regular polygon of nine sides, each of length 2 is inscribed in a circle. The radius of the circle is - **[IIT-1994]**

(A) $\text{cosec}\left(\frac{\pi}{9}\right)$ (B) $\text{cosec}\left(\frac{\pi}{3}\right)$

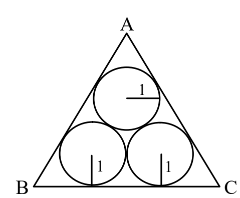
(C) $\cot\left(\frac{\pi}{9}\right)$ (D) $\tan\left(\frac{\pi}{9}\right)$

Q.14 In a triangle ABC, a : b : c = 4 : 5 : 6 . The ratio of the radius of the circumcircle to that of the incircle is- **[IIT-1996]**

(A) 16/7 (B) 7/16

(C) 16/3 (D) none of these

Q.15 In any equilateral Δ, three circles of radii one are touching to the sides given as in the figure then area of the Δ **[IIT-2005]**



(A) $6 + 4\sqrt{3}$ (B) $12 + 8\sqrt{3}$

(C) $7 + 4\sqrt{3}$ (D) $4 + \frac{7}{2}\sqrt{3}$

Q.16 If the angle A, B and C of a triangle are in an arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$ is **[IIT 2010]**

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) 1 (D) $\sqrt{3}$

Q.17 Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a, b and c denote the lengths of the sides opposite to A, B and C respectively. The value (s) of x for which $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$ is (are) **[IIT 2010]**

(A) $-(2 + \sqrt{3})$ (B) $1 + \sqrt{3}$

(C) $2 + \sqrt{3}$ (D) $4\sqrt{3}$

Numerical Response Question:

Q.18 Let ABC and ABC′ be two non-congruent triangles with sides AB = 4, AC = AC′ = $2\sqrt{2}$ and angle B = 30°. The absolute value of difference between the areas of these triangles is………….. **[IIT 2009]**

Q.19 Let PQR be a triangle of area Δ with a = 2, $b = \frac{7}{2}$ and $c = \frac{5}{2}$, where a, b and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R respectively. Then $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$ equals **[IIT 2012]**

(A) $\frac{3}{4\Delta}$ (B) $\frac{45}{4\Delta}$

(C) $\left(\frac{3}{4\Delta}\right)^2$ (D) $\left(\frac{45}{4\Delta}\right)^2$

Q.20 In a triangle PQR, P is the largest angle and $\cos P = \frac{1}{3}$. Further the incircle of the triangle touches the sides PQ, QR and RP at N, L and M respectively, such that the lengths of PN, QL and RM are consecutive even integers. Then possible length(s) of the side(s) of the triangle is (are) – **[JEE - Advance 2013]**

(A) 16 (B) 18 (C) 24 (D) 22

ANSWER KEY

LEVEL- 1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	C	C	C	A	C	D	D	C	A	B	C	C	B	B	D	C	D	A	B
Q.No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34						
Ans.	A	B	C	A	A	A	B	B	A	B	A	D	C	C						

LEVEL- 2

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Ans.	D	C	B	D	A	C	B	C	A	A	C	B	C	A	C	B

LEVEL- 3

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Ans.	C	A	A	C	A	C	D	B	C	D	D	A	B	C	B	A	A	B	C

LEVEL- 4

SECTION-A

Q.No.	1	2	3	4	5	6
Ans.	C	B	C	B	C	C

SECTION-B

1.[C] $\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a^2+b^2}{abc}$

$\frac{2bc\cos A + ac\cos B + 2ab\cos C}{abc} = \frac{a^2+b^2}{abc}$

$b^2 + c^2 - a^2 + \frac{a^2+c^2-b^2}{2} + a^2 + b^2 - c^2 = a^2 + b^2$

$\frac{a^2+c^2-b^2}{2} = a^2 - b^2$

$a^2 + c^2 - b^2 = 2a^2 - 2b^2$

$a^2 + c^2 = 2a^2 - b^2$

$c^2 + b^2 = a^2$ Hence $\angle A = \pi/2$

2.[D] $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos c}{c}$

$\Rightarrow \frac{\cos A}{k\sin A} = \frac{\cos B}{k\sin B} = \frac{\cos C}{k\sin C}$

$\Rightarrow$ cot A = cot B = cot C

$\Rightarrow$ A = B = C = 60°

$\Rightarrow$ Δ ABC is equilalateral

Area $\Delta = \frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{\sqrt{3}}{4} \times 4 = \sqrt{3}$

3.[B] Here $\alpha + \beta + \gamma = 2\pi$

A.M.$= \frac{1}{3}[\cos(\alpha + \pi/2) + \cos(\beta + \pi/2) + \cos(\gamma + \pi/2)]$

$= -\frac{1}{3}[\sin\alpha + \sin\beta + \sin\gamma]$

$= -\frac{1}{3}[2\sin\frac{\alpha+\beta}{2} + \cos\frac{\alpha-\beta}{2} + 2\sin\frac{\gamma}{2}\cos\frac{\gamma}{2}]$

$= -\frac{2}{3}\sin\frac{\gamma}{2}[\cos\frac{\alpha-\beta}{2} + \cos\frac{\alpha+\beta}{2}]$

$= -\frac{4}{3}\sin\alpha/2\,\sin\beta/2\,\sin\gamma/2$

The A.M. is least if the product $\sin\alpha/2\,\sin\beta/2\,\sin\gamma/2$ is greatest But $\sin\alpha/2\,\sin\beta/2\,\sin\gamma/2$ is greatest when $\sin\alpha/2 = \sin\beta/2 = \sin\gamma/2$

$\Rightarrow \alpha = \beta = \gamma = 120$

Thus Minimum value of A.M. is

$= -\frac{4}{3}\sin^3 60$

$= \frac{-\sqrt{3}}{2}$

4.[A] Let $\angle BAD = \alpha$ and $\angle CAD = \beta$

Then in ΔBAD

$$\frac{BD}{\sin\alpha} = \frac{AD}{\sin\pi/3}$$

$$\Rightarrow AD = \frac{BD}{\sin\alpha}\ \frac{\sqrt{3}}{2}$$

In ΔCAB $\dfrac{CD}{\sin\beta} = \dfrac{AD}{\sin\pi/4}$

$$AB = \frac{CD}{\sin\beta} \times \frac{1}{\sqrt{2}}$$

$$\frac{BD}{\sin\alpha}\sqrt{\frac{3}{2}} = \frac{CD}{\sin\beta} \times \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\sqrt{3}}{\sqrt{2}} \times \frac{BD}{CD} = \frac{\sin\alpha}{\sin\beta}$$

Or $\dfrac{\sin\alpha}{\sin\beta} = \dfrac{1}{3}\dfrac{\sqrt{3}}{2} = \dfrac{1}{\sqrt{6}}$

$$\frac{\sin\angle BAD}{\sin\angle CAB} = \frac{1}{\sqrt{6}}$$

5.[B] Let the altitude through P, Q , R be respectively α, β, γ then

$$\Delta = \frac{1}{2}\alpha P = \frac{1}{2} qr \sin P$$

$$\Rightarrow \sin P = \frac{\alpha P}{qr}$$

Similarly $\sin Q = \dfrac{Bq}{rP}$ and $\sin R = \dfrac{ry}{pq}$

Now $\dfrac{\sin P}{p} = \dfrac{\sin Q}{q} = \dfrac{\sin R}{r}$

$$\Rightarrow \frac{\alpha}{qr} = \frac{\beta}{rp} = \frac{\gamma}{pq}$$

$$\alpha ; \beta ; \gamma = \frac{1}{p} : \frac{1}{q} : \frac{1}{r}$$

$$= \frac{1}{\sin P} : \frac{1}{\sin Q} : \frac{1}{\sin R}$$

Since sin P, sin Q, sin R are in A.P.

$\Rightarrow \alpha, \beta, \gamma$ are H.P.

6.[B] In DPQR radius of circumcircle is PQ = PR

$$PQ = PR = \frac{PQ}{2\sin R} = \frac{QR}{2\sin P} = \frac{PR}{2\sin Q}$$

$$\Rightarrow \sin R = \sin Q = \frac{1}{2} \Rightarrow \angle R = \angle Q = \pi/6$$

$$\angle P = \pi - \angle R - \angle Q = 2\pi/3$$

7.[A] Since the coordinates of the centroid are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Centroid is always a rational point

8.[A] $\angle P + \angle Q + \angle R = \pi$

$P + Q = \pi/2$

$P/2 + Q/2 = \pi/4$

$\tan(P/2 + Q/2) = \tan\pi/4$

$$\frac{\tan P/2 + \tan Q/2}{1 - \tan P/2 \tan Q/2} = 1$$

$$\Rightarrow \frac{-b/a}{1 - c/a} = 1$$

$$\Rightarrow \frac{-b}{a - c} = 1$$

$a - c = -b$

$a + b = c$

9.[B] $2ac \sin\left(\dfrac{A + C - B}{2}\right)$

$$= 2ac \sin\frac{(\pi - 2B)}{2}$$

$= 2ac \cos B$

$$= 2ac\ \frac{(a^2 + c^2 - b^2)}{2ac}$$

$= a^2 + c^2 - b^2$

10.[C] $\angle A = 4x$

$\angle B = x$

$\angle C = x$

$\angle A + \angle B + \angle C = 180$

$6x = 180$

$x = 30°$

$\angle A = 120°$

$\angle B = 30°$

$\angle C = 30°$

$$\frac{a}{\sin 120} = \frac{b}{\sin 10} = \frac{c}{\sin 30} = k$$

$$a = \frac{\sqrt{3}}{2}k$$

$$b = \frac{k}{2}$$

$$c = \frac{k}{2}$$

$$2S = a + b + c = \frac{\sqrt{3}}{2}k + k$$

$$\frac{a}{2S} = \frac{\dfrac{\sqrt{3}k + 2k}{2}}{\dfrac{\sqrt{3}k}{2}} = \frac{\sqrt{3} + 2}{\sqrt{3}}\ \frac{\dfrac{\sqrt{3}\,k}{2}}{\dfrac{\sqrt{3}k + 2k}{2}} = \frac{\sqrt{3}}{2 + \sqrt{3}}$$

11.[D] Let a = x
b = x
c = 2x

$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{3x^2+4x^2-x^2}{4\sqrt{3}x^2}$

$= \frac{6x^2}{4\sqrt{3}x^2}$

$\cos A = \frac{\sqrt{3}}{2} = \cos 30$

$\cos \angle A = 30$

$\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{x^2+4x^2-3x^2}{4x^2}$

$\cos B = \frac{2x^2}{4x^2} = \frac{1}{2} = \cos 60$

B = 60°

∠C = 90°

A : B : C = 1 : 2 : 3

12.[A] $\frac{b+c}{a} = \frac{\sin B + \sin C}{\sin A}$

$= \frac{2\sin\frac{B+C}{2}\cos\frac{B-C}{2}}{2\sin A/2\cos A/2}$

$= \frac{\cos\frac{B-C}{2}}{\sin A/2}$ [B + C = π – A]

$\frac{b-c}{a} = \frac{\sin B - \sin C}{\sin A}$

$= \frac{2\cos\frac{B+C}{2}\sin\frac{B-C}{2}}{2\sin A/2\cos A/2}$

$= \frac{\sin\frac{B-C}{2}}{\cos A/2}$

13.[C] If r is the radius of inscribed circle of n side polygon then

$r = \frac{a}{2}\cot\left(\frac{\pi}{n}\right)$

Hence a = 2 and n = 9

$r = \cot\left(\frac{\pi}{9}\right)$

14.[A] a : b : c = 4 : 5 : 6
Let a = 4k
b = 5x
c = 6x

$s = \frac{15x}{2}$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{\frac{15x}{2}\left(\frac{15x}{2}-4x\right)\left(\frac{15x}{2}-5x\right)\left(\frac{15x}{2}-6x\right)}$

$= \sqrt{\frac{15x\times 7x\times 5x\times 3x}{4\times 4}}$

$= \frac{15x^2\sqrt{7}}{4}$

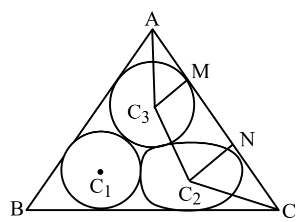
$r = \frac{\Delta}{S} = \frac{\sqrt{7}x}{2}$

$R = \frac{abc}{4\Delta} = \frac{8x}{\sqrt{7}}$

$\frac{R}{r} = \frac{16}{7}$

15.[A] $\angle C_3AM = \pi/6$

$\Rightarrow AM = C_3M \cot \pi/6 = \sqrt{3}$



Now side of the ΔABC

= AM + MN + NC

$= \sqrt{3} + 2 + \sqrt{3}$

$= 2(1 + \sqrt{3})$

Hence Area $= \frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{\sqrt{3}}{4}(1+\sqrt{3})^2$

$= 6 + 4\sqrt{3}$

16.[D] A, B, C A.P.

2B = A + C

A + B + C = 180° ⇒ 3B = 180 ⇒ B = 60°

$\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A =$

$= \frac{a}{c}2\sin C\cos C + \frac{c}{a}2\sin A\cos A$

$= \frac{a'\cos C}{R} + \frac{C\cos A}{R} \Rightarrow \frac{a\cos c + c\cos A}{R}$

$= \left\{\because \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{2R}\right.$

$= \frac{a\cos C + c\cos A}{R} = \frac{b}{R}$

$= \frac{2R\sin B}{R} = 2\sin 60 = \sqrt{3}$

17.[B] Using cosine rule for $\angle C$

$$\frac{\sqrt{3}}{2} = \frac{(x^2+x+1)^2+(x^2-1)^2-(2x+1)^2}{2(x^2+x+1)(x^2-1)}$$

$$\Rightarrow \sqrt{3} = \frac{2x^2+2x-1}{x^2+x+1}$$

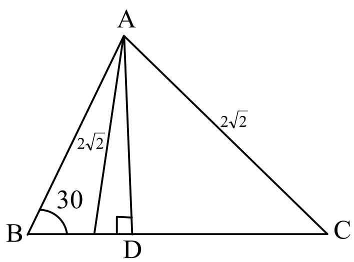
$$\Rightarrow (\sqrt{3}-2)x^2 + (\sqrt{3}-2)x + (\sqrt{3}+1) = 0$$

$$\Rightarrow x = \frac{(2-\sqrt{3}) \pm \sqrt{3}}{2(\sqrt{3}-2)}$$

$$\Rightarrow x = -(2+\sqrt{3}),\ 1+\sqrt{3}$$

$$x = 1 + \sqrt{3}$$

18.[4] $\cos 30 = \dfrac{a^2+16-8}{2 \times a \times 4}$



$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{a^2+8}{2}$$

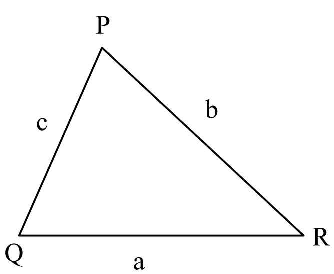
$$\Rightarrow a^2 - 4\sqrt{3}\,a + 8 = 0$$

$$\Rightarrow a_1 + a_2 = 4\sqrt{3}$$

$$|a_1 - a_2| = 4$$

$$|\Delta_1 - \Delta_2| = \frac{1}{2} \times 4 \sin 30 \times 4 = 4$$

19.[C]



$$\left.\begin{aligned} a &= 2 \\ b &= 7/2 \\ c &= 5/2 \end{aligned}\right\} \Rightarrow s = 4$$

$$\frac{2\sin P - 2\sin P\cos P}{2\sin P + 2\sin P\cos P} = \frac{1-\cos P}{1+\cos P}$$

$$\Delta = \sqrt{4.2.\frac{1}{2}.\frac{3}{2}} = \sqrt{6}$$

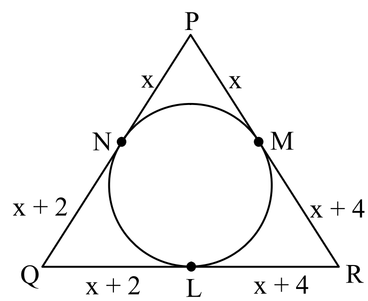
$$= \tan^2\frac{P}{2}$$

$$= \left(\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}\right)^2$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)}{4(2)} = \frac{3}{32}$$

$$= \left(\frac{9}{16.6}\right) = \left(\frac{3}{4\Delta}\right)^2$$

20. [B,D]



$$\cos P = \frac{(2x+2)^2+(2x+4)^2-(2x+6)^2}{2(2x+2)(2x+4)}$$

$$\Rightarrow \frac{1}{3} = \frac{4x^2-16}{2(4x^2+12x+8)}$$

$$\Rightarrow 8x^2 + 24x + 16 = 12x^2 - 48$$

$$\Rightarrow 4x^2 - 24x - 64 = 0$$

$$\Rightarrow x^2 - 6x - 16 = 0$$

$$\Rightarrow (x-8)(x+2) = 0$$

$$\Rightarrow x = 8$$

So the sides of triangles are, 18, 20, 22.